ALEKS® Practice Test Module B #1

College Algebra / Math 121 B – GARAGE (Dr. Vasan)

Student Name/ID:	Key S(mart) Student

1. Answer the questions below about the quadratic function.

$f(x) = -2x^2 + 16x - 34$

(a) Coefficient of x^2 is -2 (negative)
Therefore graph looks like
It has a maximum

(a)	Does the function have a minimum or maximum value? Maximum
(b)	What is the function's minimum or maximum value? $y = -2$
(c)	Where does the minimum or maximum value occur? $x = 4$

(~~b~~) $x = -\frac{b}{2a}$
for vertex
$= \frac{-16}{2(-2)} = 4$

(c) $x = 4$

(b) $y = -2(4)^2 + 16(4) - 34$
$= -32 + 64 - 34$
$= -2$

2. Write the quadratic function in the form $f(x) = a(x-h)^2 + k$.

Then, give the vertex of its graph.

$f(x) = 3x^2 - 18x + 31$

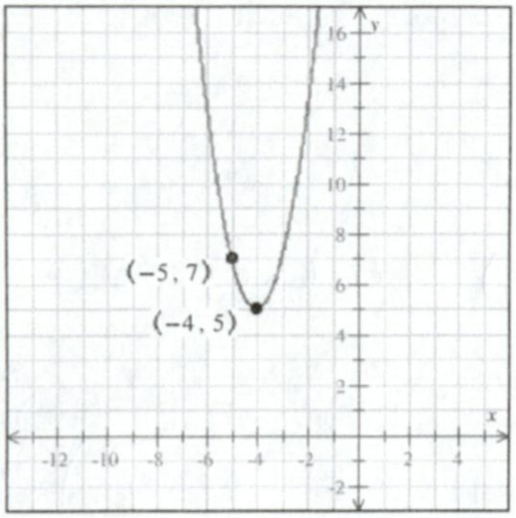
$= 3(x^2 - 6x) + 31$ $\qquad \frac{-6}{2} = -3$

$= 3(x^2 - 6x + 9) + 31 - 27$ $\qquad (-3)^2 = 9$

$= 3(x-3)^2 + 4$

Vertex $= (3, 4)$ Answer

3. Find the equation of the quadratic function f whose graph is shown below.



$y = a(x-h)^2 + k$

$(h,k) = (-4,5)$

$y = a(x+4)^2 + 5$

When $x = -5$, $y = 7$

$7 = a(-5+4)^2 + 5$

$= a(-1)^2 + 5$

$7 = a + 5$

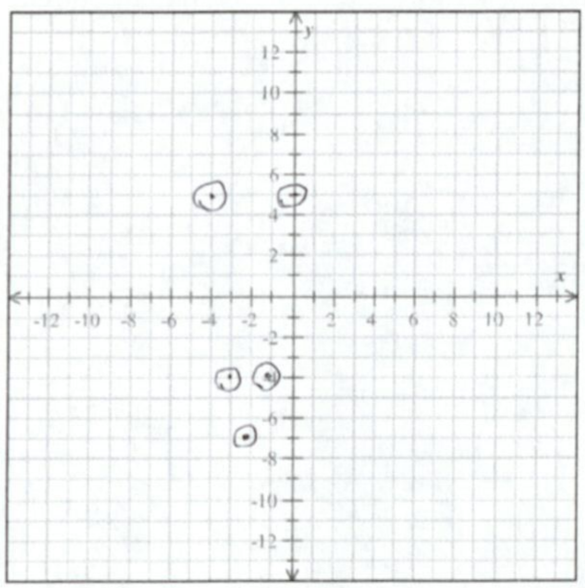
$-5 \quad -5$

$2 = a$

Answer $\boxed{y = 2(x+4)^2 + 5}$

4. Graph the parabola.

$y = 3x^2 + 12x + 5$



$y = 3(x^2 + 4x) + 5$

$\frac{4}{2} = 2$

$(2)^2 = 4$

$y = 3(x^2 + 4x + 4) + 5 - 12$

$= 3(x+2)^2 - 7$

Vertex $= (-2, -7)$

x	y	
−4	5	
−3	−4	
−2	−7	vertex
−1	−4	
0	5	

5. Find all real zeros of the function.

$$2x(x-3)^2(x-4)^2 = 0$$

$2x = 0$ $\quad x = 0$

$x - 3 = 0$ $\quad +3 \;\; +3$ $\quad x = 3$

$x - 4 = 0$ $\quad +4 \;\; +4$ $\quad x = 4$

$$f(x) = 2x(x-3)^2(x-4)^2$$

Zeros are 0, 3, 4

If there is more than one answer, separate them with commas.

6. Find a polynomial $f(x)$ of degree 4 that has the following zeros.

$$-2, 1, -6, 0$$

$x = -2$ $\quad +2 \;\; +2$ $\quad x + 2 = 0$

$x = 1$ $\quad -1 \;\; -1$ $\quad x - 1 = 0$

$x = -6$ $\quad +6 \;\; +6$ $\quad x + 6 = 0$

$x = 0$ $\quad x = 0$

Leave your answer in factored form. Multiply the four factors!

Answer $x(x+2)(x-1)(x+6) = 0$

7. Consider the following polynomial functions.

(a) $f(x) = -3(x+1)^2(x+3)^2$

(b) $g(x) = x^3 - x^2 - 6x$

Choose the graph of each function from the choices below.

(a) $f(x) = -3(x+1)^2(x+3)^2$

Find the zeros

$-3(x+1)^2(x+3)^2 = 0$

$x + 1 = 0$ $\quad x = -1$

$x + 3 = 0$ $\quad x = -3$

Multiplicity of 2 for each zero.

Therefore $f(x)$ touches the x-axis at $x = -1$ and $x = -3$

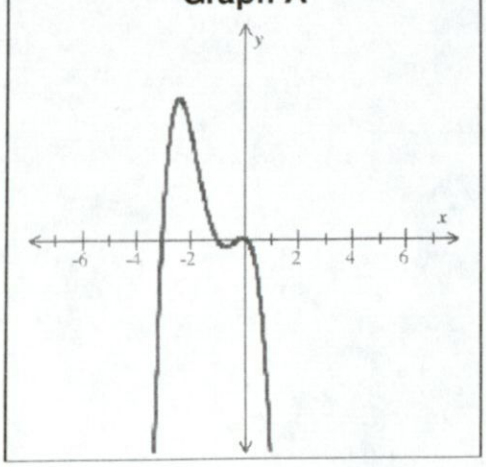
Choice is between A and F

$f(x)$ is even with leading negative coefficient

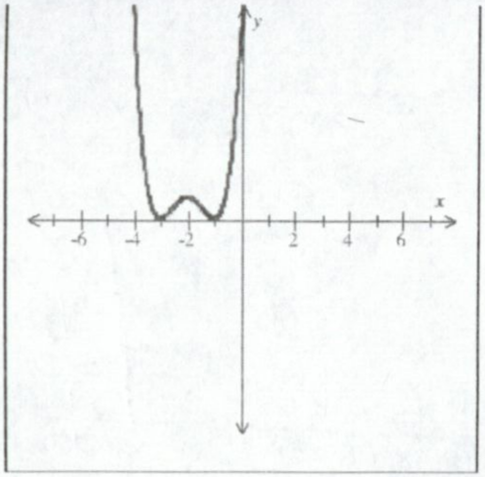
Therefore it should fall both on the left and on the right

$\boxed{\text{Answer is F}}$

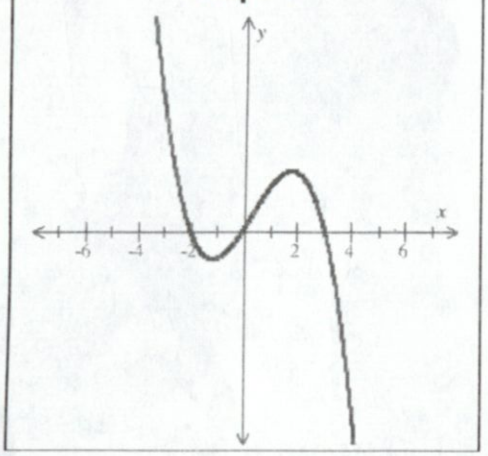
Graph A



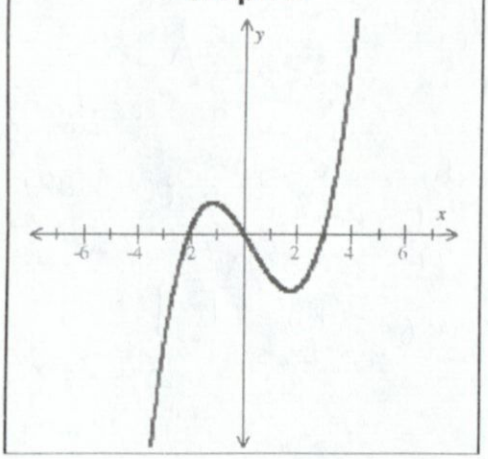
Graph B



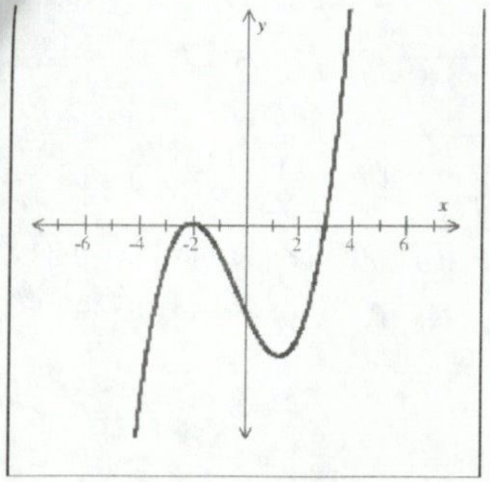
Graph C



Graph D



Graph E



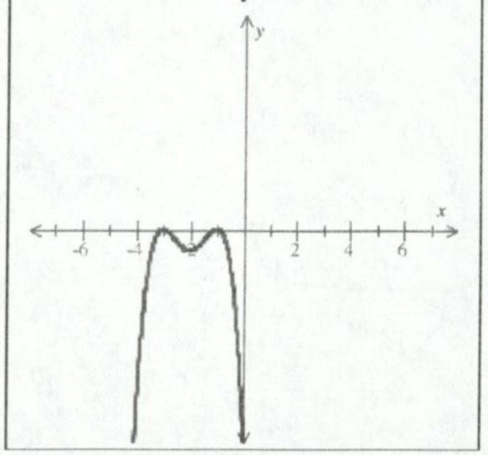
(b) $g(x) = x^3 - x^2 - 6x$
$= x(x^2 - x - 6)$
$= x(x-3)(x+2)$

Zeros of $g(x)$
$x(x-3)(x+2) = 0$
$x = 0, \; x = 3, \; x = -2$
Multiplicity is 1 each.
Therefore $g(x)$ crosses the x-axis at $0, 3, -2$
Choices are C and D.

$g(x)$ is odd and has positive leading coefficient.
Therefore it must fall on the left and rise on the right

$\boxed{\text{Answer is D}}$

Graph F



Which is the graph of $f(x) = -3(x+1)^2(x+3)^2$?	F
Which is the graph of $g(x) = x^3 - x^2 - 6x$?	D

8. Find all x-intercepts and y-intercepts of the graph of the function.

$$f(x) = 2x^3 + 2x^2 - 18x - 18$$

If there is more than one answer, separate them with commas.

x-intercept(s): $-3, 3, -1$

y-intercept(s): -18

Factor by grouping
$2x^2(x+1) - 18(x+1)$
$= (2x^2 - 18)(x+1)$
$= 2(x^2 - 9)(x+1)$
$= 2(x+3)(x-3)(x+1) = 0$
$x = -3, \; x = 3, \; x = -1$ are the x-intercepts

To find y-intercept, put $x = 0$
$2(0)^3 + 2(0)^2 - 18(0) - 18$
$= -18$

9. Graph all vertical and horizontal asymptotes of the function.

$$f(x) = \frac{-9x-1}{2x^2-2x-12}$$

V.A. $2x^2-2x-12=0$

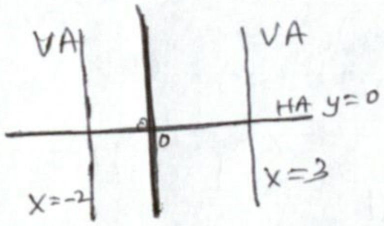
$2(x^2-x-6)=0$

$2(x-3)(x+2)=0$

V.A @ $x=3$ and $x=-2$

H.A Degree of numerator < Degree of denominator

Therefore HA $y=0$



10. Graph the rational function $f(x) = \frac{2x+3}{x+3}$.

To graph the function, draw the horizontal and vertical asymptotes (if any) and plot at least two points on each piece of the graph.

VA

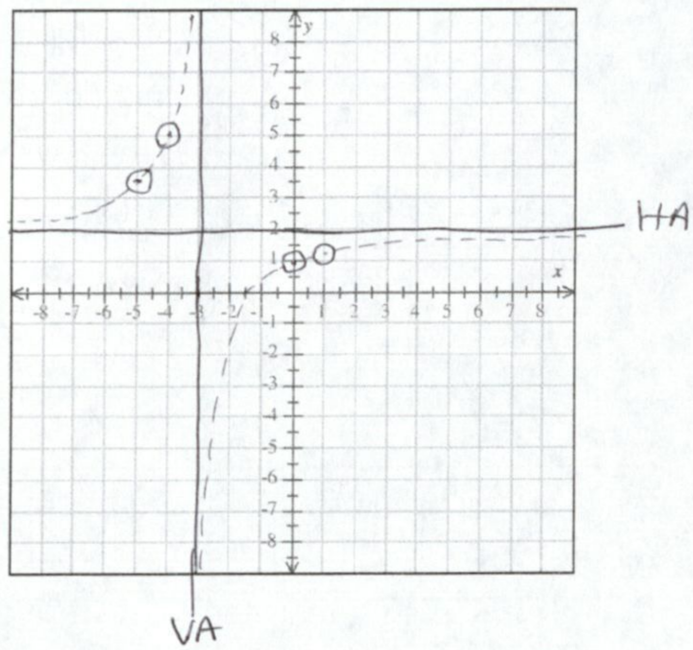
$x+3=0$

$x=-3$

HA $\frac{2}{1}=2$

Calculate x, y points

x	y
0	1
1	$\frac{5}{4}$
-4	5
-5	$\frac{7}{2}$



11. Solve the inequality.

$$x^3+12x > -8x^2$$

Rewrite as

$x^3+8x^2+12x > 0$

Write your answer as an interval or union of intervals.

or $x(x^2+8x+12) > 0$

or $x(x+2)(x+6) > 0$

$x=0$, $x=-2$, $x=-6$ are points where the sign changes

Number line

	-6		-2		0	
Sign of $x+6$	– –		+ + + +		+ + + + +	
Sign of $x+2$	– –		– – – –		+ + + + +	
Sign of x	– –		– – – –		– –	+ + +
OVERALL SIGN	–		+		–	+

Answer:

$(-6,-2) \cup (0,\infty)$

Rewrite as

$$\frac{x-1}{-(x-6)} > 0$$

Multiply by -1

$$\frac{x-1}{x-6} < 0$$

12. Solve the following inequality.

$$\frac{x-1}{-x+6} > 0$$

Write your answer using interval notation.

		1		6	
Sign of $x-1$	$- -$		$+ + + + +$		$+ + +$
Sign of $x-6$	$-$		$- - - -$		$+ + +$
Overall sign	$+$		$-$		$+$

Answer $(1, 6)$

13. A supply company manufactures copy machines. The unit cost C (the cost in dollars to make each copy machine) depends on the number of machines made. If x machines are made, then the unit cost is given by the function $C(x) = 0.5x^2 - 170x + 25{,}850$. What is the minimum unit cost?

Do not round your answer.

x value of vertex $= \frac{-b}{2a}$

$= \frac{-(-170)}{2(0.5)} = 170$

We need the y-value of vertex

$C(170) = 0.5(170)^2 - 170(170) + 25{,}850$ Answer

Which works out to \$11,400 if you had a calculator.

14. Graph the parabola.

$$y = -3x^2 + 6x + 4$$

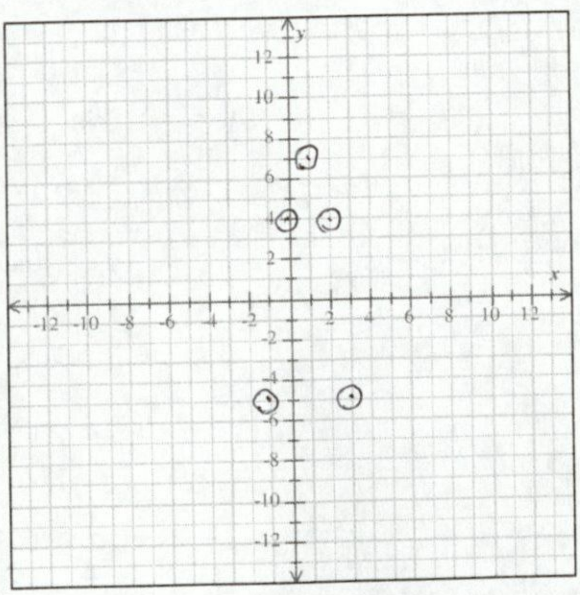
$= -3(x^2 - 2x) + 4$

$= -3(x^2 - 2x + 1) + 4 + 3$

$= -3(x-1)^2 + 7$

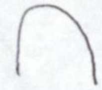
Vertex is $(1, 7)$

	x	y
	-1	-5
	0	4
vertex	1	7
	2	4
	3	-5



15. Find the range of the quadratic function.

$$f(x) = -x^2 + 8x - 18$$

This parabola would look like this 

Vertex $x = \frac{-b}{2a} = \frac{-8}{2(-1)} = 4$

Write your answer using interval notation.

$y = -(4)^2 + 8(4) - 18$
$= -16 + 32 - 18 = -2$

Therefore -2 is the maximum y-value

Answer: $(-\infty, -2]$

16. Write the quadratic function in the form $f(x) = a(x-h)^2 + k$.

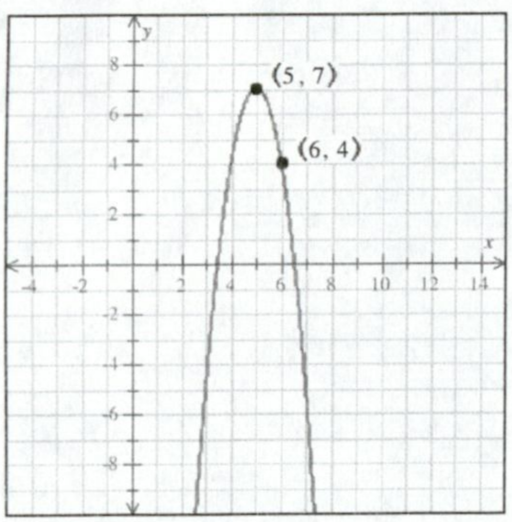
Then, give the vertex of its graph.

$$f(x) = -3x^2 + 12x - 13$$

$= -3(x^2 - 4x) - 13$
$= -3(x^2 - 4x + 4) - 13 + 12$
$= -3(x-2)^2 - 1$ Answer

Vertex: $(2, -1)$ Answer

17. Find the equation of the quadratic function f whose graph is shown below.



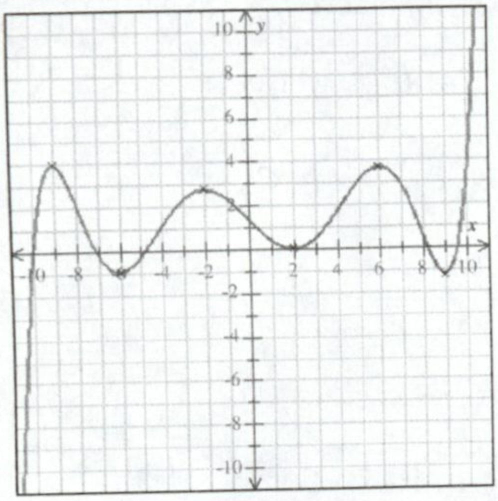
$y = a(x-h)^2 + k$
$(h, k) = (5, 7)$
$y = a(x-5)^2 + 7$
When $x = 6$, $y = 4$
$4 = a(6-5)^2 + 7$
$4 = a(1)^2 + 7$
$4 = a + 7$
$-7 \quad -7$
$-3 = a$

$y = -3(x-5)^2 + 7$ Answer

18. Choose the end behavior of the graph of each polynomial function.

(a) $f(x) = x^5 - 3x^3 - 2x^2 + 2$ {(a) Rises, (b) Falls} to the left and {(a) rises, (b) falls} to the right.	Odd a Positive leading coefficient (LC) Falls to the left, rises to the right
(b) $f(x) = 3x^3 + 6x^2 + 9x + 4$ {(a) Rises, (b) Falls} to the left and {(a) rises, (b) falls} to the right.	Odd, Positive LC b Falls to the left, rises to the right
(c) $f(x) = -x(x-3)(5x+2)$ {(a) Rises, (b) Falls} to the left and {(a) rises, (b) falls} to the right.	Odd, Negative LC c Rises to the left and falls to the right

19. Below is the graph of a polynomial function f with real coefficients. Use the graph to answer the following questions about f. All local extrema of f are shown in the graph.



(a) The function f is increasing over which intervals? Choose all that apply.

$(-\infty, -9)$ $(-6, -2)$ $(2, 6)$

$(6, 9)$ $(2, 9)$ $(9, \infty)$

$(-\infty, -9)$ $(-6, -2)$ $(2, 6)$ $(9, \infty)$

(b) The function f has local maxima at which x-values? If there is more than one value, separate them with commas. Maximum occurs at $x = -9$, $x = -2$, $x = 6$

(c) What is the sign of the leading coefficient of f?

Positive Negative Not enough information

Falls to the left, Rises to the right
Therefore it is odd function
with positive leading coefficient

(d) Which of the following is a possibility for the degree of f? Choose all that apply.

4 5 6 7 8 9

# of turning points = 6
Add 1. Therefore the degree can be 7 or 9

20. Graph all vertical and horizontal asymptotes of the function.

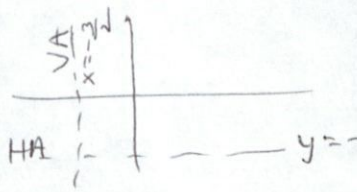
$$f(x) = \frac{-10x+13}{4x+6}$$

V.A

$4x + 6 = 0$
$-6 \quad -6$
$4x = -6$
$x = -\frac{6}{4} = \boxed{-\frac{3}{2}}$

H.A. Degrees of numerator and denominator are equal

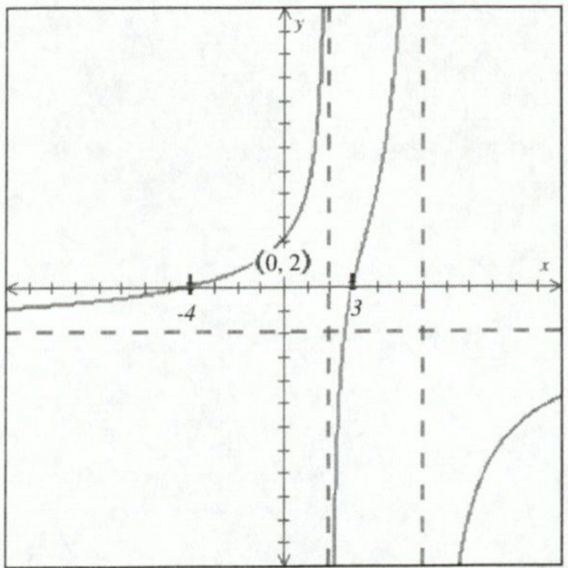
Therefore HA $= \frac{-10}{4} = \boxed{\frac{-5}{2}}$



21. The figure below shows the graph of a rational function f.
It has vertical asymptotes $x = 2$ and $x = 6$, and horizontal asymptote $y = -2$.
The graph has x-intercepts 3 and -4, and it passes through the point $(0, 2)$.

The equation for $f(x)$ has one of the five forms shown below.
Choose the appropriate form for $f(x)$, and then write the equation.
You can assume that $f(x)$ is in simplest form.

2 VAs
- Therfoe it can be C, D or E
- It has a HA = -2
- It cannot be C or D
- It has to be E



A ○ $f(x) = \dfrac{a}{x-b}$

B ○ $f(x) = \dfrac{a(x-b)}{x-c}$

C ○ $f(x) = \dfrac{a}{(x-b)(x-c)}$

D ○ $f(x) = \dfrac{a(x-b)}{(x-c)(x-d)}$

E ○ $f(x) = \dfrac{a(x-b)(x-c)}{(x-d)(x-e)}$

Intercepts go into the numerater

VAs go into the denominator

$a = -2$

Answer is

$$\frac{-2(x-3)(x+4)}{(x-2)(x-6)}$$

22. Solve.

$$4|x-9| - 5 \le 11$$

$$4|x-9| - 5 \le 11$$
$$\quad +5 \quad +5$$
$$4|x-9| \le 16$$
$$|x-9| \le 4$$
$$-4 \le x-9 \le 4$$
$$+9 \quad +9 \quad +9$$
$$5 \le x \le 13$$ Answer